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COMPUTER PROGRAM FOR ROUND BUOYANT JETS INTO STRATIFIED AMBIENT ENVIRONMENTS

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March 1969

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"Computer Program For Round Buoyant Jets Into Stratified Ambient Environments"

by

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p. 3, line 4	$1/\lambda^2$ is the turbulent Schmidt number
p. 9, line 10	the entrainment coefficient (α), the spreading ratio (λ), and the kinematic viscosity (ν)
p. 14, line 27	is in use at this laboratory

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I. Summary

The gross behavior of an inclined round turbulent buoyant jet in a stratified ambient environment is determined by quadrature of the governing differential equations. The FORTRAN IV (level G) language is used for the program, which has been run on an IBM 360/75 digital computer. The essential input to the problem includes the location of the jet; the initial values of jet velocity, jet diameter, and angle of inclination; the density of the discharged fluid; and the density profile of the ambient environment. The density profile may have any gravitationally stable shape. It is provided as input to the program by supplying the density at arbitrarily selected points (which best fit the actual profile) and calculated as linear between these points. The output consists of the jet trajectory in rectangular coordinates; the nominal width; the centerline velocity, density difference, and dilution ratio. All of these parameters are printed out at uniform intervals along the jet trajectory. Calculations are stopped at the maximum height of rise or greatest depth of sinking of the jet or at any predetermined vertical or horizontal coordinate.

The program is applicable to several limiting cases of round turbulent jets. Horizontal, vertical, and inclined jets ($-90° \leq \theta_o \leq 90°$) with initially positive or negative buoyancies can be handled in stratified or uniform ambient environments. The jet may be placed at any elevation in the ambient environment. Simple momentum jets can be handled, although simple plumes are not amenable to this technique.

For cases of linear density profiles in the environment a variety of numerical computer solutions have been presented in generalized non-dimensional coordinates by Fan and Brooks (6). However, for arbitrary non-linear density profiles, generalized non-dimensional solutions are not feasible; it is for this reason that the computer program is being made available for case by case solutions.

II. Basic Assumptions and Equations

A schematic diagram of a round buoyant jet beginning at the origin at an angle θ_o with the horizontal is shown in Figure 1. The gross behavior of such a jet in a stagnant linear density profile has been analyzed by Fan (1). The general assumptions of his analysis are:

1. The fluids are incompressible.

2. The Boussinesq assumption requires that variations of density throughout the flow field are small relative to the reference density chosen.

3. The density of the fluid is assumed to be a linear function of either salt concentration or temperature for the range of variation.

4. The flow is fully turbulent.

5. The longitudinal turbulent transport is small relative to longitudinal convection.

6. The curvature of the jet trajectory is small.

7. The velocity profiles are similar at all cross sections normal to the jet trajectory. Also, the profiles for buoyancy and concentration of a tracer are assumed similar. The analyses apply only to the zone of established flow and the similarity is met by using axisymmetric Gaussian profiles, as follows:

Velocity:
$$u^*(s,r) = u(s)e^{-r^2/b^2} \tag{1}$$

Buoyancy:
$$\frac{\rho_a^*(s,r) - \rho^*(s,r)}{\rho_o} = \frac{\rho_a(s) - \rho(s)}{\rho_o} e^{-r^2/(\lambda b)^2} \tag{2}$$

Tracer:
$$c^*(r,s) = c(s)e^{-r^2/(\lambda b)^2} \tag{3}$$

The u*, ρ_a*, ρ*, c* are the velocity, ambient density, density in the jet, and the concentration in the jet, respectively, and are related to the centerline values u(s), ρ_a(s), c(s) by the Gaussian form. The nominal half width for the jet is $\sqrt{2}$ b(= 2σ), and λ^2 is the turbulent Schmidt number, assumed constant. Also $\rho_o = \rho_a(0)$.

8. The entrainment relation or volume flux derivative is given by the equation:

$$dQ/ds = 2\pi\alpha ub \tag{4}$$

where α is a coefficient of entrainment, assumed constant.

Using these assumptions, the conservation equations for mass, momentum, and buoyancy were evaluated using the integral technique of Morton, Taylor, and Turner (2). Together with the geometric relations, this method yields the following equations (Fan, 1):

Continuity: $$\frac{d}{ds}(ub^2) = 2\alpha ub \tag{5}$$

x-Momentum: $$\frac{d}{ds}\left(\frac{u^2b^2}{2}\cos\theta\right) = 0 \tag{6}$$

y-Momentum: $$\frac{d}{ds}\left(\frac{u^2b^2}{2}\sin\theta\right) = g\lambda^2 b^2 \frac{(\rho_a - \rho)}{\rho_o} \tag{7}$$

Buoyancy: $$\frac{d}{ds}\left[ub^2(\rho_a - \rho)\right] = \frac{1+\lambda^2}{\lambda^2} b^2 u \frac{d\rho_a}{ds} \tag{8}$$

Tracer: $$\frac{d}{ds}(c\,ub^2) = 0 \tag{9}$$

Geometry: $$\frac{dx}{ds} = \cos\theta \tag{10}$$

Geometry $$\frac{dy}{ds} = \sin\theta \tag{11}$$

where the seven variables u, b, θ, c, $(\rho_a - \rho)$, x, and y are functions of s only.

For the case of the constant ambient density gradient, these equations have been written in nondimensional form and solved by quadrature (see Fan (1, 6) for details). For the case of an arbitrary ambient density profile considered here, the term $\frac{d\rho_a}{ds}$ is considered to be the local ambient density gradient which now varies with height y. Since the ambient density gradient is variable with y and along the trajectory (s), equations 5-11 will be solved by computer in dimensional form. By simplifying and combining the conservation equations, the following set of first-order ordinary differential equations is obtained:

$$\frac{du}{ds} = \frac{2g\lambda^2}{u} \frac{(\rho_a - \rho)}{\rho_o} \sin\theta - \frac{2u\alpha}{b} \tag{12}$$

$$\frac{db}{ds} = 2\alpha - \frac{b}{u^2} g\lambda^2 \frac{(\rho_a - \rho)}{\rho_o} \sin\theta \tag{13}$$

$$\frac{d\theta}{ds} = \frac{2g\lambda^2}{u^2} \frac{(\rho_a - \rho)}{\rho_o} \cos\theta \tag{14}$$

$$\frac{d(\rho_a - \rho)}{ds} = \frac{1 + \lambda^2}{\lambda^2} \sin\theta \frac{d\rho_a}{dy} - \frac{2\alpha}{b} (\rho_a - \rho) \tag{15}$$

$$\frac{dx}{ds} = \cos\theta \tag{16}$$

$$\frac{dy}{ds} = \sin\theta \tag{17}$$

$$c\, u\, b^2 = \text{constant} = c_o\, u_o\, b_o^2 \tag{18}$$

Examination of these equations has shown that they are valid for the case of an initially positive or negative density difference, (buoyant or sinking jet). In addition, they reduce to the proper forms for the case of a uniform ambient density environment

$\left(\frac{d\rho_a}{dy} = 0\right)$ and for the case of a simple momentum jet $\left(\frac{d\rho_a}{dy} = 0 \text{ and } (\rho_a - \rho) = 0\right)$. Since no singularities exist for either a vertical or a horizontal jet, the angle of initial inclination may range from -90° to 90°. However, these equations are not valid for the case of a simple plume which requires that the initial momentum flux be zero.

III. Method of Solution

The set of first-order differential equations describing the jet behavior can be solved using a FORTRAN compatible subroutine. For the program developed here, the subroutine DEQ/Differential Equation Solver-360 was employed. This subroutine is on file at the Booth Computing Center of California Institute of Technology. It uses the Runge-Kutta-Gill method of integration with an automatic control of the truncation error. Although the size of the integration step is fixed by the user, this subroutine will subdivide the interval in an attempt to preserve a specified number of significant figures locally throughout the integration. In this program, the allowable truncation error has been set to preserve five significant figures locally. If this error is exceeded, notification is provided in an output statement. In general, if the user chooses an integration step in accordance with the guidelines provided in a section below, the notification of excessive error will occur only when the jet has reached the maximum height of rise. Physically, at that point the fluid will have negative buoyancy and fall out of a jet pattern and mathematically, the equations are no longer valid and the solutions meaningless.

While solution of the differential equations is made relatively straightforward by the use of the DEQ subroutine, account must be taken of the local ambient density gradient term which varies along the jet trajectory. To accomplish this, the depth of the

ambient fluid is divided in segments, each of height Δy, as shown in Figure 2. For convenience Δy was chosen equal to the integration step size along the jet trajectory. The ambient density profile is specified so that the density is known at each multiple of Δy throughout the depth (as explained below). The local ambient density gradient within each layer, or height Δy, is calculated as the difference in density between the top and bottom of the layer divided by Δy. The program is constructed so that the local ambient density gradient term in the equations is that which is appropriate for the horizontal layer through which the calculation is proceeding and that it changes as the trajectory moves on to the next layer. Although the jet may proceed part way into layer N+1 using the density gradient of layer N, this error cannot exist over more than the distance Δy along the trajectory before it is corrected. By selecting sufficiently small Δy this localized error will have little effect on the gross behavior of the jet.

The ambient density profile is established by providing as input the densities at selected heights above the origin which are multiples of Δy. Since Δy is very small relative to the total depth or expected height of rise of the jet, the user has the capability of providing a density value at practically any height within the accuracy of his knowledge of that value and height. Density values must be provided at the bottom of the profile and the water surface. Between these values, the density values may be provided at any heights in accordance with the above explanation. The density values are calculated at each multiple of Δy by considering linear gradients to exist between the values which have been provided by the user. Thus, any gravitationally stable density profile can be closely approximated by this method.

The equations describing the gross jet behavior are valid and the calculation begins in the zone of established flow. An adequate similarity solution for the zone of flow establishment is not available for a buoyant jet in a stratified ambient fluid. Albertson et al. (3) have described the zone of flow establishment for a simple

momentum jet. For the lack of more complete information of this region in a buoyant jet, their results are applied to provide the initial width of the jet and centerline concentration of a tracer at the beginning of the zone of established flow (a distance of 6.2 jet diameters from the actual source). This program begins the calculation of the jet trajectory at the beginning of the zone of established flow with the inclination of the jet from the horizontal unchanged from that of the actual source (see Figure 3). The initial buoyancy of the jet is considered to have no effect on the geometry in this zone. Thus, in cases in which the ratio of the length of the trajectory to the jet diameter becomes small (of the order of 20) and the initial buoyancy is large, this program may provide unreliable results because of these simplifying assumptions.

IV. Use of the Computer Program (Table 1)

This program determines the gross behavior of a buoyant jet in physical dimensions. The data input to the program must therefore be in compatible units, with the exception of the density data which may be in any self-consistent units (such as specific gravity or sigma units).

The program in FORTRAN (level G) language is presented in Table 1. The input of data is divided into four statements as indicated by the four READ statements in the program. The purpose and use of each statement with its components will be explained separately. The FORMAT statement for each READ statement is found immediately following it in the program and is in standard form.

1. READ (5, 101) VEL, DIAM, THETAO, DENDIF

This requires the input of the initial mean jet velocity, (VEL), the initial jet diameter (DIAM), the initial inclination of the jet from the horizontal in degrees (THETAO), and the initial density difference (DENDIF), respectively. The initial jet diameter may be the diameter of the discharge tube or of the contracted jet diameter in the case of an orifice or nozzle. Hence the product (VEL)$\cdot \frac{\pi}{4}$ (DIAM)^2 should simply be the initial discharge.

This program adjusts the initial density difference, supplied as input, to account for the effect of the zone of flow establishment by multiplying it by the factor $\frac{1+\lambda^3}{2\lambda^3}$. For the case of a zero ambient density gradient in the zone of flow establishment, this adjustment assures that the correct initial buoyancy flux is provided. For the case of a nonzero ambient density gradient in this region, this adjustment is not strictly correct as the changing of the density of the entrained ambient fluid is neglected. Hence, the program treats the zone of flow establishment as one of uniform

ambient density. The input difference is defined as:

$$\left(\rho_o - \rho_d\right)$$

where ρ_o = ambient density at the point of discharge (or at the end of the zone of flow establishment, whichever is chosen for the ambient density in the zone of flow establishment).

ρ_d = density of the discharged fluid.

2. READ (5, 200) GRAV, ALPHA, SCHMD, VISC

This requires the input of the acceleration due to gravity (g), the entrainment coefficient (λ), and the kinematic viscosity (ν), respectively. Fan's study (1) recommends that an entrainment coefficient of 0.082 be used for a buoyant jet (or plume) and that an entrainment coefficient of 0.057 be used for a simple momentum jet. He suggests that the parameter λ be set equal to 1.16 based on the studies of Morton (4) and Rouse et. al. (5).

3. READ (5, 199) JETEL, MAXY, NOLAYR, DELTAY, XLIMIT, YLIMIT

Before proceeding further one must select an appropriate vertical spacing, Δy, and the density values which will be used to specify the ambient density profile. It is important to choose Δy small enough to assure integration of the differential equations without exceeding the specified computational error and terminating the calculation. It is convenient to choose Δy to be some number of which multiples are easily recognized (e.g. 0.5 ft, 1 cm., etc. depending on the scale of the problem). A guide to the choice of Δy is as follows:

a. Choose Δy such that, $100 \leq \frac{y_t}{\Delta y} \leq 300$

where y_t = estimated terminal height of rise of the jet above its initial elevation.

b. The program allows $\left(\frac{d}{\Delta y}\right)_{max} = 998$

where d = total depth of fluid

As shown in Figure 2, each elevation and layer is assigned a fixed point number (an integer without a decimal point). The calculations start at elevation JETEL and the density values begin at elevation 1. (If geometric corrections for the zone of flow establishment are made, the calculations begin 6.2 cos θ_o jet diameters horizontally and 6.2 sin θ_o jet diameters vertically away from the actual point of discharge.) The program requires that the total height of fluid above this point be assigned a fixed point number. The equation relating the actual dimensional height to its fixed point dimensionless height is:

$$\text{LAYER (a fixed point no.)} = \frac{y}{\Delta y} + 1$$

where y = the actual dimensional height rounded off to an integer number of Δy's.

e.g. If y = 15.4 feet and Δy = 0.5 feet,

$$\text{LAYER} = \frac{15.4}{.5} + 1 = 30.8 + 1 = 32$$

The READ statement above requires the input of the fixed point dimensionless elevation of the jet (JETEL), the fixed point dimensionless height of the fluid surface or maximum height to which a density value is assigned (MAXY), the fixed point number of density values which will be read in (NOLAYR), the value of Δy (DELTAY), the maximum horizontal distance the calculation will be allowed to reach (XLMIT), and the maximum vertical distance the calculation will be allowed to reach (YLIMIT) respectively. The last two distances provide an ultimate stop for the calculation in the event that the maximum height of rise is the total depth of fluid. For the case of a simple momentum jet they stop the calculation at the desired coordinates at which information is required, as theoretically, the jet trajectory is infinitely long.

4. READ (5,201) LAYER, AMDEN (LAYER)

This statement requires the input of ordered pairs of fixed point dimensionless heights and density values to define the ambient density profile. The fixed point dimensionless height is calculated as described above.

As many data cards of this form as are required to approximate the density profile are supplied. (This number should agree with that stated in the proceeding READ statement.) At least two cards are required, stating the bottom and surface heights and respective density values.

The output of this program is displayed for an example problem presented in the next section. It consists of a title and a print out of the first three sets of input data. The program calculates the densimetric Froude number and the Reynolds number of the jet as follows:

$$\text{Densimetric Froude No.} = \frac{U}{\sqrt{g\left(\frac{\rho_o - \rho_d}{\rho_o}\right) D}}$$

$$\text{Reynolds No.} = \frac{UD}{\nu}$$

where: U = discharge velocity

D = jet diameter

$\frac{\rho_o - \rho_d}{\rho_o}$ = dimensionless initial density difference

ν = kinematic viscosity

g = acceleration of gravity

The calculated values of these parameters are printed as output. The Froude number should exceed 2 and the Reynolds number exceed 5000 to assure the validity of the assumptions underlying the jet calculations.

The remainder of the output concerns the calculation of the jet behavior. After each step of integration (a distance Δy along the jet trajectory), the following results are printed in tabular form: the distance along the trajectory, the centerline velocity, the nominal jet width, the centerline density difference, the angle of inclination from the horizontal in degrees, the vertical coordinate, the horizontal coordinate, the centerline dilution ratio, the

ambient density value and the layer number associated with the layer in which the calculation has been made. All values except the dilution ratio are in the physical dimensions of the input data. The nominal jet width is defined as 4σ of the Gaussian distribution. The centerline dilution ratio is the ratio of the centerline concentration of a conservative tracer substance at the given coordinates to that of the initial discharge. The initial value of this parameter is $\frac{2\lambda^2}{1+\lambda^2}$ because of the correction for the zone of flow establishment. It should be noted again that the calculations begin at the zone of established flow and that $6.2\ D \cos\theta_o$ and $6.2 \sin\theta_o$ should be added to the x- and y- coordinates, respectively, if a geometric correction is made for the zone of flow establishment.

Finally, when the calculation has been terminated, a statement is printed out giving an explanation for the termination. For instance, the maximum height of rise has been reached or one of the limiting coordinates specified in the input has been reached. Often in the case of a vertical buoyant jet, the local allowable error of integration is exceeded because the vertical velocity becomes extremely small just prior to the maximum height of rise. In that case, the calculation is terminated because of the error, but the coordinates also indicate the maximum height of rise quite well. However, if the velocity just before termination is not small, one should suspect that the error has been exceeded because Δy is too large and another calculation should be attempted with a smaller value of Δy.

V. An Example Problem

A buoyant jet is discharged horizontally from a tube into a stratified ambient fluid as shown in Figure 4. The initial data are as follows:

location of jet	1.0 ft. above the bottom
discharge angle	$\theta_o = 0.0°$
discharge velocity	$U_o = 8.0$ ft/sec
jet diameter	$D = 0.33$ ft.

density of discharge	$\rho_d = 0.99350$ g/ml
ambient density at discharge elevation	$\rho_o = 1.02460$ g/ml
density difference	$\rho_o - \rho_d = 0.03110$ g/ml
depth of fluid	d = 37.7 ft.

The following constants are used:

gravity	$g = 32.2$ ft/sec^2
entrainment coefficient	$\alpha = 0.082$
square root of the Schmidt number	$\lambda = 1.16$
kinematic viscosity	$\nu = 1.00 \times 10^{-5}$ ft^2/sec

The ambient density profile in this case is approximated in simple fashion by three straight line segments (specified below). Hence, the number of density values to be supplied as input is four. The vertical spacing, Δ_y, is chosen to be 0.2 ft. If one estimates that the maximum height of rise might be near the bottom of the break in the profile at about 24 ft., the value of $\frac{y_t}{\Delta y}$ is 115. Since this is within the range suggested, $\Delta y = 0.2$ ft. will be used to calculate the dimensionless heights for the input data.

Dimensionless total height $= \frac{d}{\Delta y} + 1 = \frac{37.7}{0.2} + 1 \doteq 190 =$ MAXY

(note that the total depth was rounded off to an integer number of Δy's)

Dimensionless jet elev. $= \frac{y}{\Delta y} + 1 = \frac{1.0}{0.2} + 1 = 6 =$ JETEL

The calculation is limited in the vertical direction by YLIMIT = 37.8 and in the horizontal direction by XLIMIT = 50.0. This will prohibit calculation to extend beyond these distances in the event that the maximum height of rise is not reached before either of these coordinates.

The density data, the dimensionless height (LAYER) and d density value are supplied as follows:

at $y = 0$ $\quad$ LAYER $= \frac{y}{\Delta y} + 1 = 0 + 1 = 1$ $\quad \rho = 1.02460$

$y = 23.6$ ft. LAYER $= \frac{23.6}{.2} + 1 = 118 + 1 = 119$ $\rho = 1.02452$

$y = 29.1$ ft. LAYER $= \frac{29.1}{.2} + 1 = 145.5 + 1 = 147$ $\rho = 1.02272$

$y = 37.7$ ft. LAYER $= \frac{37.7}{.2} + 1 = 188.5 + 1 = 190$ $\rho = 1.02272$

Note that dimensionless heights have again been rounded off to an integer number of Δy's.

The necessary data has been determined and is found at the end of the program presented in Table 1 in the proper FORMAT arrangement.

The program with the input data for this problem has been run on the IBM 360/75 computer at the California Institute of Technology. The output and results are shown in Table 2. The jet densimetric Froude number is 14.1 and the jet Reynolds number is 2.64×10^5. The results indicate that the maximum height of rise of the centerline of the jet trajectory is 29.7 feet The dilution ratio at this point is 49.7. The fluid in the jet will fall back to its level of neutral buoyancy near the elevation 31.0 feet as shown by the change in sign of the centerline density difference.

Note that the computations in this example start from a given elevation y = 1.0 ft, in terms of the y-coordinates which are fixed in relation to the density profile. Thus it is easy to repeat the calculations for various possible levels of the discharge point, by changing only the input JETEL, without complete recasting of the input data for the density profile.

The user can make minor adjustments in the program for machine plotting of results as desired. Such a plotting subroutine is in use t this laboratory, but has not been included herein because of its lack of generality for other computing centers.

Acknowledgment

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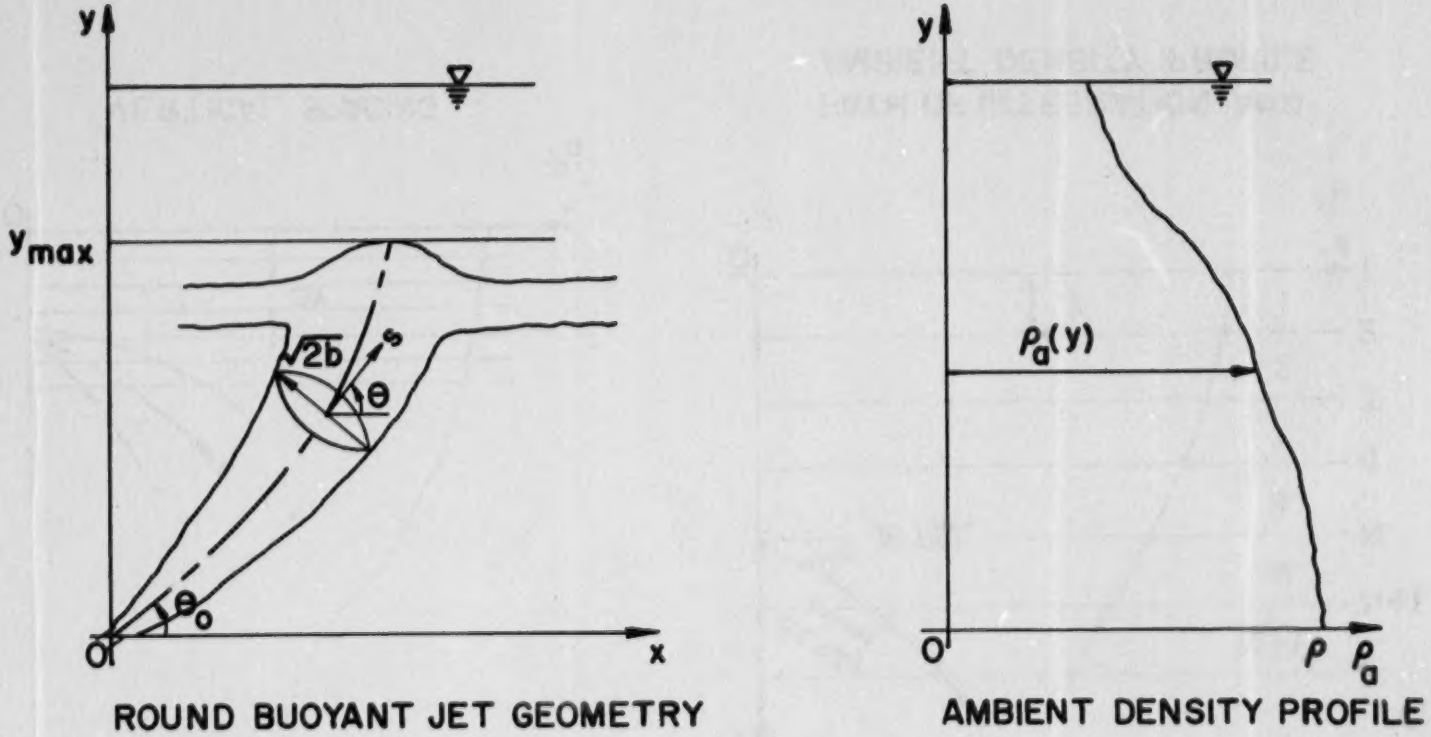

Figure 1. Schematic diagram of a round buoyant jet in a stagnant environment with an arbitrary density profile.

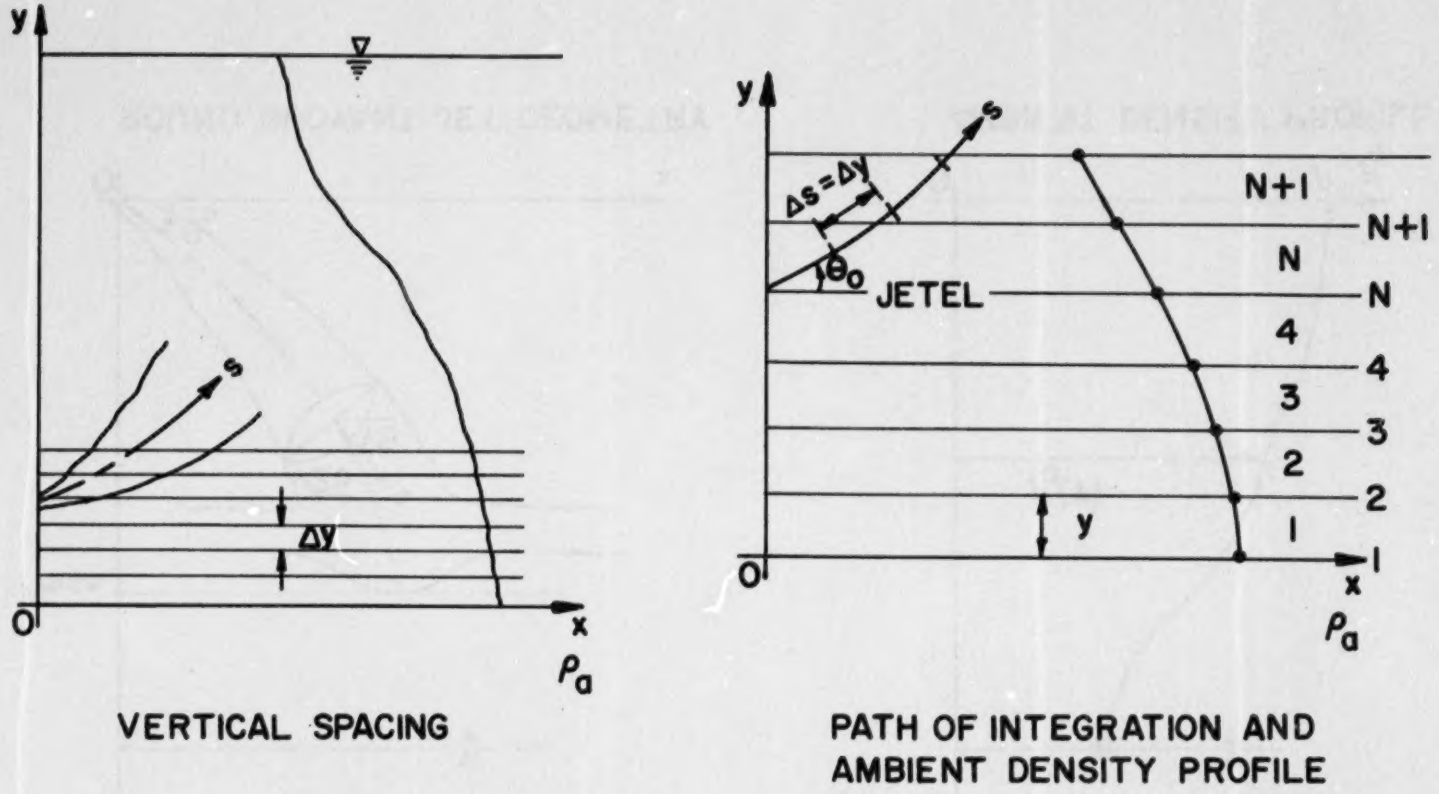

Figure 2. Schematic diagram of vertical spacing for jet trajectory integration and approximation of ambient density profile.

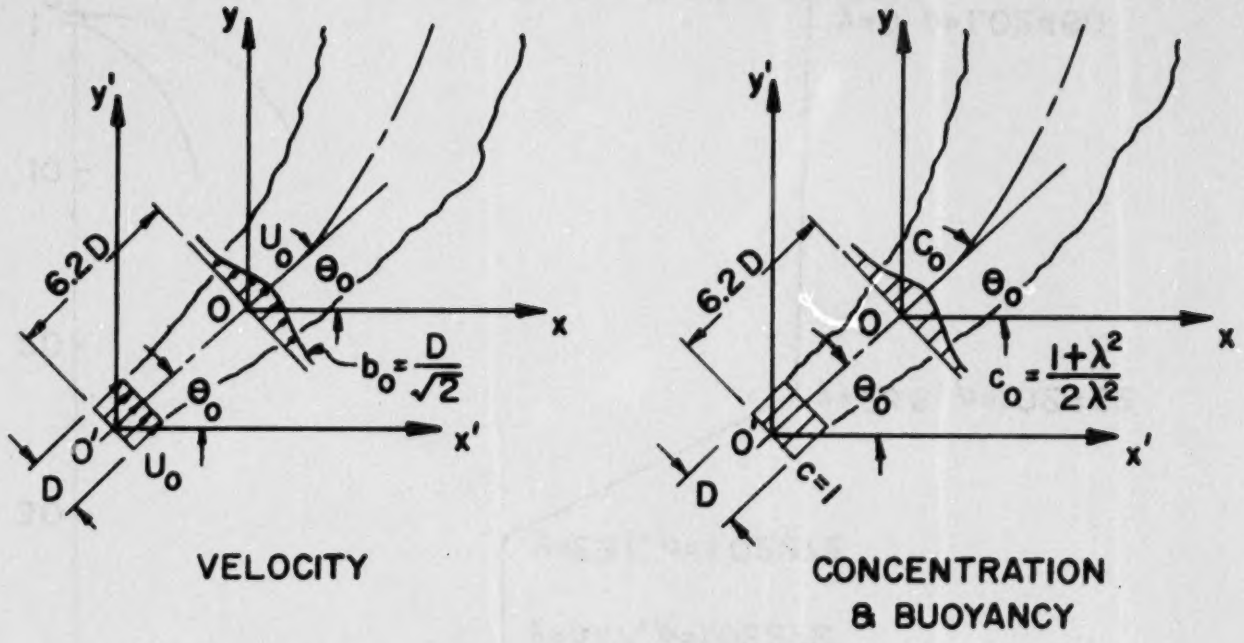

Figure 3. Zone of flow establishment for an inclined buoyant jet discharging at the origin O'. The program begins calculation at the origin O.

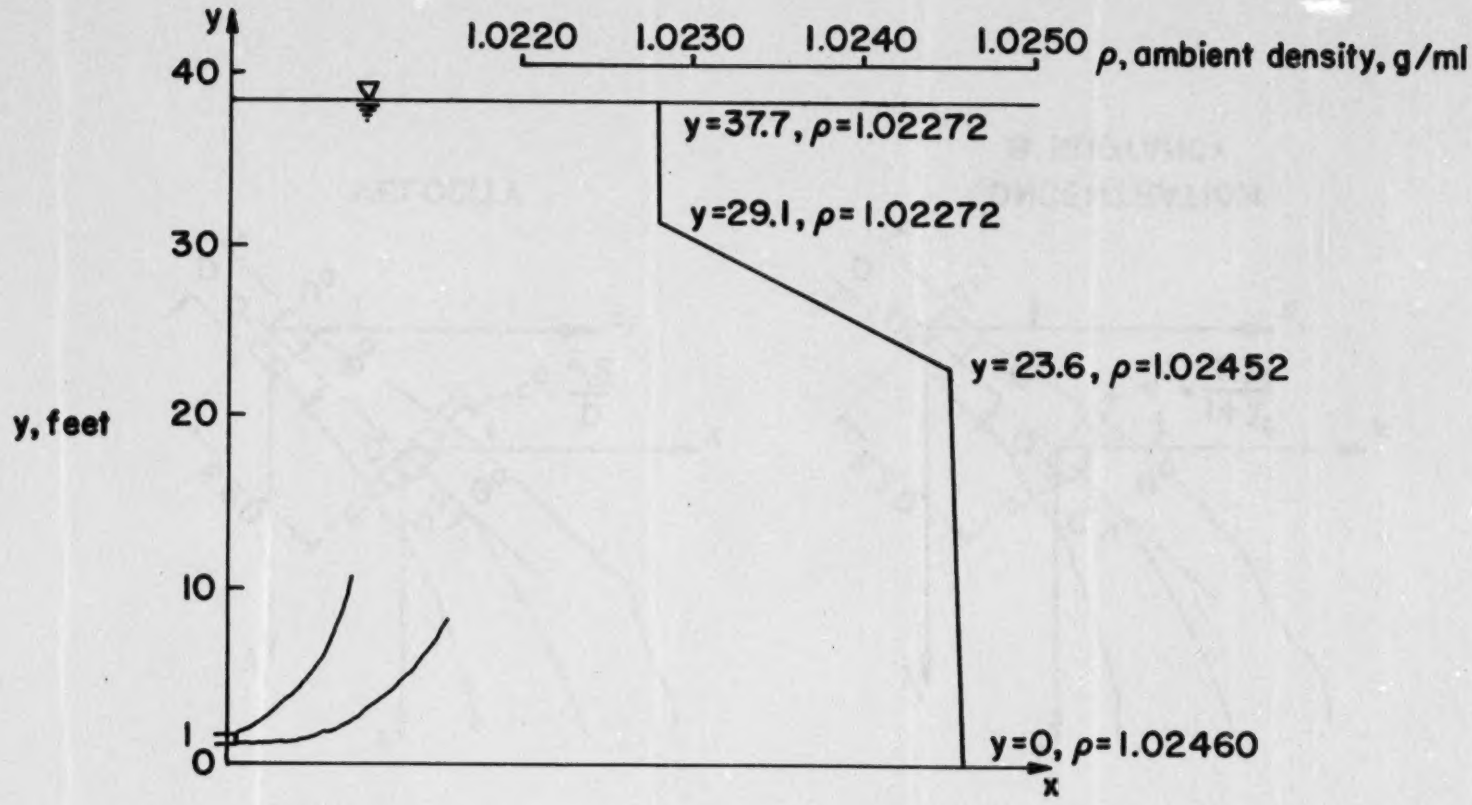

Figure 4. Density profile for example problem.

COMPUTER PROGRAM

```
//NBJET   JOB  (51401,JDD,CE),DITMARS,MSGLEVEL=1
//STEP1  EXEC   FORTGCLG,REGION.GO=100K
//FORT.SYSIN DD *
C INCLINED,BUOYANT,ROUND JET IN AN ARBITRARY DENSITY PROFILE
C DENSITY VALUES MAY BE READ IN AT ANY ELEVATION WHICH IS AN INTEGER
C   NUMBER OF DELTAYS (AT LEAST AT THE BOTTOM AND SURFACE) AND A LINEAR
C   PROFILE IS CALCULATED BETWEEN THE GIVEN POINTS.
C VEL = INITIAL CENTERLINE JET VELOCITY
C DIAM = DIAMETER OF THE JET INITIALLY
C THETAO = INITIAL INCLINATION OF THE JET FROM THE HORIZONTAL - DEGREES
C DENDIF = INITIAL DENSITY DIFFERENCE AT THE CENTERLINE
C GRAV = ACCELERATION DUE TO GRAVITY
C ALPHA = ENTRAINMENT COEFFICIENT, .082 FOR BUOYANT JET, .057 FOR
C   MOMENTUM JET IS RECOMMENDED
C SCHMD  = SQRT OF THE TURBULENT SCHMIDT NUMBER   (1.16 RECOMMENDED)
C VISC = KINEMATIC VISCOSITY
C JETEL = DIMENSIONLESS JET LOCATION(ELEVATION)
C MAXY = NUMBER OF SURFACE LAYER  =(TOTAL DEPTH/DELTAY) + 1
C NOLAYR = NUMBER OF DENSITY DATA VALUES TO BE READ IN
C DELTAY = VERTICAL SPACING FOR DENSITY DATA AND JET INTEGRATION
C XLIMIT = HORIZONTAL LIMITATION-CALCULATION STOPS AT X=XLIMIT, IF NOT
C   BEFORE
C YLIMIT = VERTICAL LIMITATION-CALCULATION STOPS AT Y=YLIMIT, IF NOT
C   BEFORE
C LAYER = NUMBER ASSIGNED TO A DELTAY SEGMENT,(HEIGHT/DELTAY) + 1
C AMDEN(M) = AMBIENT DENSITY AT LEVEL LAYER=M
C DENGRD(M) = LOCAL DENSITY GRADIENT IN LAYER=M
C Y(1) = CENTERLINE VELOCITY
C Y(2) = JET WIDTH B = SQRT(2) SIGMA
C Y(3) = CENTERLINE DENSITY DIFFERENCE
C Y(4) = THETA, INCLINATION OF THE JET FROM THE HORIZONTAL,RADIANS
C Y(5) = X, HORIZONTAL DISTANCE
C Y(6) = Y, VERTICAL DISTANCE
C T = DISTANCE ALONG THE JET TRAJECTORY
C DEQ = A SUBROUTINE TO SOLVE FIRST ORDER D.E.'S USING A RUNGE-KUTTA
C   TECHNIQUE
C WIDTH = NOMINAL JET WIDTH = 4 SIGMA
C DILUT = CENTERLINE DILUTION RATIO
      DOUBLE PRECISION AMDEN,DENGRD
      DIMENSION Y(6),YDOT(6),AMDEN(1000),DENGRD(1000),YCOOR(1000)
      READ(5,101) VEL,DIAM,THETAO,DENDIF
  101 FORMAT(3F10.5,F10.7)
      READ(5,200) GRAV,ALPHA,SCHMD,VISC
  200 FORMAT(3F10.5,E10.6)
      READ(5,199) JETEL, MAXY,NOLAYR,DELTAY,XLIMIT,YLIMIT
  199 FORMAT(3I4,3F10.4)
      WRITE(6,300)
      WRITE(6,150) VEL,DIAM,THETAO,DENDIF
      WRITE(6,203) GRAV,ALPHA,SCHMD,VISC
      WRITE(6,204) JETEL,MAXY,NOLAYR,DELTAY,XLIMIT,YLIMIT
      AJ = JETEL - 1
      AJET = AJ*DELTAY
      DO 49 LAYER = 1,MAXY
      YMK = LAYER - JETEL
```

```
      YCOOR(LAYER) = YMK*DELTAY
   49 AMDEN(LAYER) = 0.0
      N = 1
   52 IF(NOLAYR-N) 51,50,50
   50 READ(5,201) LAYER, AMDEN(LAYER)
  201 FORMAT(I4,F10.7)
      N = N+1
      GO TO 52
   51 LN1 = 1
      LN2 = 2
   54 IF(AMDEN(LN2). GT. 0.0) GO TO 53
      LN2 = LN2 + 1
      GO TO 54
   53 DELTA = LN2 - LN1
      LN11 = LN1 + 1
      LN21 = LN2 - 1
      DO 55 M = LN1,LN21
   55 DENGRD(M) = (AMDEN(LN2)-AMDEN(LN1))/(DELTA*DELTAY)
      IF((LN2-LN1)-1) 59,59,60
   60 DO 56 J = LN11,LN21
   56 AMDEN(J)=AMDEN(J-1)+(DENGRD(J-1)*DELTAY)
   59 IF(LN2-MAXY) 57,58,58
   57 LN1 = LN2
      LN2 = LN2 + 1
      GO TO 54
   58 CONTINUE
      RHOO = AMDEN(JETEL)
      IF(DENDIF - 0.0) 70,71,72
   70 DNDFN = -DENDIF
      FROUDE = VEL/SQRT(GRAV*DNDFN*DIAM/RHOO)
      WRITE(6,401) FROUDE
      GO TO 73
   71 WRITE(6,402)
      GO TO 73
   72 FROUDE = VEL/SQRT(GRAV*DENDIF*DIAM/RHOO)
      WRITE(6,401)FROUDE
   73 REYN = VEL*DIAM/VISC
      WRITE(6,403) REYN
      WRITE(6,301)
      COEFF = GRAV*(SCHMD**2)/RHOO
      COEFF2= 4.0 *(SCHMD**2)/((1.+(SCHMD**2))*(VEL*(DIAM**2)))
      Y(1) = VEL
      Y(2) =DIAM/1.414214
      Y(3) = DENDIF*(1.+(SCHMD**2))/(2.*(SCHMD**2))
      Y(4) = THETAO*3.1415927/180.
      Y(5) = 0.0
      Y(6) = 0.0
      T = 0.0
      NTIME = 1
      IF(THETAO .GT. 0.0) GO TO 2
      IF(THETAO .LT. 0.0) GO TO 4
      IF(THETAO .EQ. 0.0) GO TO 3
    3 IF(DENDIF .GT. 0.0) GO TO 2
      IF(DENDIF .LT. 0.0) GO TO 4
      IF(DENDIF .EQ. 0.0) GO TO 2
    2 M = JETEL
      MY = JETEL + 1
      GO TO 5
    4 M = JETEL - 1
      MY = JETEL - 1
    5 N = 1
```

```
      CALL DEQ(K,6,T,Y,YDOT,DELTAY,5.E-6)
  100 GO TO (10,20,20,30),K
   10 YDOT(1) = (-2.*ALPHA*Y(1)/Y(2))+(2.*COEFF*Y(3)*SIN(Y(4))/Y(1))
      YDOT(2) = (2.*ALPHA)-(COEFF*Y(2)*Y(3)*SIN(Y(4))/(Y(1)**2))
      YDOT(3) = ((1.+(SCHMD**2))*DENGRD(M)*SIN(Y(4))/(SCHMD**2))-(2.*
     1ALPHA*Y(3)/Y(2))
      YDOT(4) = (2.*COEFF)*Y(3)*COS(Y(4))/(Y(1)**2)
      YDOT(5) = COS(Y(4))
      YDOT(6) = SIN(Y(4))
      CALL DEQ2    (&100, &130)
   20 WIDTH = 2.828428*Y(2)
      DILUT = COEFF2*Y(1)*(Y(2)**2)
      YVERT = Y(6) + AJET
      THETA = Y(4)*180./3.1415927
      WRITE(6,400) T,Y(1),WIDTH,Y(3),THETA,Y(5),YVERT,DILUT,AMDEN(M),M
      N = N + 1
      IF(N-50) 75,75,76
   76 WRITE(6,405)
      N = 1
   75 CONTINUE
      IF(THETAO .GT. 0.0) GO TO 900
      IF(THETAO .LT. 0.0) GO TO 902
      IF(THETAO .EQ. 0.0) GO TO 901
  900 IF(DENDIF .GT. 0.0) GO TO 903
      IF(DENDIF .LT. 0.0) GO TO 904
      IF(DENDIF .EQ. 0.0) GO TO 903
  903 GO TO 21
  904 IF(0.0-Y(4)) 905,905,906
  906 IF(NTIME-1) 930,930,931
  930 MY = MY - 1
  931 NTIME = 2
      GO TO 22
  905 IF(1-NTIME) 907,908,908
  908 GO TO 22
  907 GO TO 24
  902 IF(DENDIF .GT. 0.0) GO TO 909
      IF(DENDIF .LT. 0.0) GO TO 910
      IF(DENDIF .EQ. 0.0) GO TO 910
  910 GO TO 211
  909 IF(0.0-Y(4)) 911,912,912
  911 IF(NTIME - 1) 932,932,933
  932 MY = MY + 1
  933 NTIME = 2
      GO TO 22
  912 IF(1-NTIME) 913,914,914
  914 GO TO 22
  913 GO TO 23
  901 IF(DENDIF .GT. 0.0) GO TO 915
      IF(DENDIF .LT. 0.0) GO TO 916
      IF(DENDIF .EQ. 0.0) GO TO 915
  915 GO TO 21
  916 GO TO 211
   21 IF(0.0 - (Y(1)*SIN(Y(4)))) 22,22,23
  211 IF(0.0 - (Y(1)*SIN(Y(4)))) 24, 22,22
   22 IF(0.0-Y(4)) 920,925,921
  925 IF(DENDIF - 0.0) 921,920,920
  920 IF(Y(6)-YCOOR(MY)) 31,26,26
   26 M = M + 1
      MY = MY + 1
      GO TO 31
  921 IF(Y(6)-YCOOR(MY)) 261,261,31
```

```
  261 M = M - 1
      MY = MY - 1
   31 IF(XLIMIT - Y(5)) 35,32,32
   32 IF(YLIMIT - YVERT) 36,331,331
  331 IF(YVERT-0.0) 341,33,33
   33 IF(MAXY - M) 34,34,27
   27 CALL DEQ1 (&100)
  130 CONTINUE
      GO TO 75
   23 WRITE(6,501)
      STOP
   24 WRITE(6,507)
      STOP
   35 WRITE(6,502)
      STOP
   36 WRITE(6,503)
      STOP
   30 WRITE(6,500)
      WRITE(6,504)
      STOP
   34 WRITE(6,505)
      STOP
  341 WRITE(6,506)
      STOP
  150 FORMAT(1HO,5X,4HVEL=,F10.5,5X,5HDIAM=,F10.5,5X,7HTHETAO=,F10.5,5X,
     17HDENDIF=,F10.7)
  203 FORMAT(1HO,5X,5HGRAV=,F10.5,5X,6HALPHA=,F10.5,5X, 6HSCHMD=,F10.5,5
     1X,10HVISCOSITY=,1PE12.5)
  204 FORMAT(1HO,5X, 6HJETEL=,I4,5X,5HMAXY=,I4,5X,7HNOLAYR=,I4,5X,7HDELT
     1AY=,F10.4,5X,7HXLIMIT=,F10.4,5X,7HYLIMIT=,F10.4)
  300 FORMAT(1H1,58HINCLINED,BUOYANT,ROUND JET IN AN ARBITRARY DENSITY P
     1ROFILE)
  301 FORMAT(1HO,4X,1HS,7X,8HVELOCITY,8X,5HWIDTH,9X,6HDENDIF,8X,5HTHETA,
     17X,7HX COORD,7X,7HY COORD,4X,8HDILUTION,6X,5HAMDEN,7X,1HM)
  400 FORMAT(1X,F7.2,3X,F10.5,4X,F10.5,4X,1PE12.5,4X,0PF7.2,4X,0PF10.5,4
     1X,0PF10.5,3X,0PF7.2,4X,0PF10.7,3X,0PI4)
  401 FORMAT(1HO,28HJET DENSIMETRIC FROUDE NO. =,F10.4)
  402 FORMAT(1HO,38HJET DENSIMETRIC FROUDE NO. IS INFINITE)
  403 FORMAT(1HO,18HJET REYNOLDS NO. =,1PE12.5)
  405 FORMAT(1H1,3X,1HS,7X,8HVELOCITY,8X,5HWIDTH,9X,6HDENDIF,8X,6HTHETAO
     1,6X,7HX COORD,7X,7HY COORD,4X,8HDILUTION,6X,5HAMDEN,7X,1HM)
  501 FORMAT(1HO,39HMAXIMUM HEIGHT OF RISE HAS BEEN REACHED)
  502 FORMAT(1HO,23HXLIMIT HAS BEEN REACHED)
  503 FORMAT(1HO,23HYLIMIT HAS BEEN REACHED)
  500 FORMAT(1HO,98HTHE ALLOWABLE ERROR IN INTEGRATING THE DIFF. EQS. HA
     1S BEEN EXCEEDED - PROBABLY BECAUSE THE MAXIMUM)
  504 FORMAT(66H HEIGHT OF RISE OR GREATEST DEPTH OF SINKING HAS JUST BE
     1EN REACHED)
  505 FORMAT(1HO,41HMAXY - THE WATER SURFACE HAS BEEN REACHED)
  506 FORMAT(1HO,27HTHE BOTTOM HAS BEEN REACHED)
  507 FORMAT(1HO,42HGREATEST DEPTH OF SINKING HAS BEEN REACHED)
      END
/*
//GO.SYSIN DD *
   8.0          .33        0.0       0.0310929
  32.2         0.082       1.16     .100000E-4
   6 190    4     0.2        50.0        37.8
   1 1.02460
 119 1.02452
 147 1.02272
 190 1.02272
```

INCLINED,BUOYANT,ROUND JET IN AN ARBITRARY DENSITY PROFILE

Table 2. Results for Example Program

VEL= 8.00000 DIAM= 0.33000 THETAO= 0.0 DENDIF= 0.0311000

GRAV= 32.20000 ALPHA= 0.08200 SCHMD= 1.16000 VISCOSITY= 1.000000E-05

JETEL= 6 MAXY= 190 NOLAYR= 4 DELTAY= 0.2000 XLIMIT= 50.0000 YLIMIT= 37.8000

JET DENSIMETRIC FROUDE NO. = 14.0864

JET REYNOLDS NO. = 2.640000E 05

S	VELOCITY	WIDTH	DENDIF	THETA	X COORD	Y COORD	DILUTION	AMDEN	M
0.0	8.00000	0.66000	2.71062E-02	0.0	0.0	1.00000	1.15	1.0245966	6
0.20	7.81427	0.75276	2.37656E-02	0.44	0.20000	1.00075	1.31	1.0245966	6
0.40	6.24530	0.84549	2.11579E-02	0.94	0.39998	1.00313	1.47	1.0245966	6
0.60	5.62891	0.93817	1.90658E-02	1.49	0.59994	1.00735	1.63	1.0245966	6
0.80	5.12406	1.03078	1.73500E-02	2.10	0.79984	1.01361	1.79	1.0245966	6
1.00	4.70325	1.12328	1.59172E-02	2.77	0.99966	1.02209	1.95	1.0245966	6
1.20	4.34739	1.21565	1.47026E-02	3.50	1.19936	1.03300	2.12	1.0245966	6
1.40	4.04282	1.30784	1.36598E-02	4.28	1.39890	1.04654	2.28	1.0245966	6
1.60	3.77948	1.39980	1.27547E-02	5.11	1.59823	1.06289	2.44	1.0245966	6
1.80	3.54984	1.49148	1.19615E-02	6.00	1.79729	1.08223	2.60	1.0245966	6
2.00	3.34810	1.58282	1.12607E-02	6.94	1.99601	1.10475	2.76	1.0245966	6
2.20	3.16978	1.67375	1.06367E-02	7.93	2.19433	1.13061	2.92	1.0245966	6
2.40	3.01132	1.76420	1.00776E-02	8.97	2.39216	1.15999	3.09	1.0245966	6
2.60	2.86988	1.85410	9.57357E-03	10.06	2.58941	1.19304	3.25	1.0245966	6
2.80	2.74314	1.94335	9.11677E-03	11.19	2.78598	1.22990	3.41	1.0245959	7
3.00	2.62922	2.03189	8.70073E-03	12.36	2.98177	1.27070	3.57	1.0245959	7
3.20	2.52654	2.11961	8.32012E-03	13.57	3.17667	1.31556	3.74	1.0245959	7
3.40	2.43379	2.20644	7.97050E-03	14.82	3.37056	1.36459	3.90	1.0245959	7
3.60	2.34987	2.29228	7.64811E-03	16.09	3.56333	1.41787	4.07	1.0245959	7
3.80	2.27381	2.37707	7.34981E-03	17.39	3.75485	1.47547	4.23	1.0245953	8
4.00	2.20482	2.46072	7.07288E-03	18.72	3.94500	1.53746	4.40	1.0245953	8
4.20	2.14217	2.54316	6.81502E-03	20.06	4.13365	1.60386	4.56	1.0245953	8
4.40	2.08526	2.62433	6.57424E-03	21.42	4.32068	1.67468	4.73	1.0245946	9
4.60	2.03353	2.70417	6.34881E-03	22.79	4.50598	1.74994	4.90	1.0245946	9
4.80	1.98649	2.78266	6.13724E-03	24.16	4.68942	1.82962	5.06	1.0245946	9
5.00	1.94372	2.85974	5.93821E-03	25.54	4.87089	1.91367	5.23	1.0245939	10
5.20	1.90481	2.93542	5.75058E-03	26.91	5.05030	2.00205	5.40	1.0245939	10
5.40	1.86943	3.00967	5.57334E-03	28.28	5.22754	2.09469	5.58	1.0245932	11
5.60	1.83724	3.08250	5.40559E-03	29.63	5.40254	2.19151	5.75	1.0245932	11
5.80	1.80796	3.15394	5.24654E-03	30.97	5.57520	2.29243	5.92	1.0245932	11
6.00	1.78132	3.22399	5.09550E-03	32.30	5.74548	2.39734	6.10	1.0245925	12
6.20	1.75708	3.29270	4.95183E-03	33.60	5.91330	2.50613	6.27	1.0245925	12
6.40	1.73501	3.36011	4.81497E-03	34.89	6.07861	2.61868	6.45	1.0245919	13
6.60	1.71490	3.42626	4.68444E-03	36.15	6.24139	2.73488	6.63	1.0245912	14
6.80	1.69659	3.49122	4.55976E-03	37.38	6.40161	2.85458	6.81	1.0245912	14
7.00	1.67988	3.55503	4.44054E-03	38.59	6.55924	2.97768	6.99	1.0245905	15
7.20	1.66464	3.61777	4.32641E-03	39.76	6.71427	3.10402	7.17	1.0245905	15
7.40	1.65070	3.67948	4.21703E-03	40.91	6.86671	3.23348	7.36	1.0245898	16
7.60	1.63796	3.74025	4.11211E-03	42.03	7.01656	3.36594	7.54	1.0245892	17
7.80	1.62628	3.80012	4.01137E-03	43.12	7.16383	3.50126	7.73	1.0245892	17
8.00	1.61555	3.85917	3.91456E-03	44.18	7.30854	3.63931	7.92	1.0245885	18
8.20	1.60569	3.91746	3.82144E-03	45.20	7.45072	3.77997	8.11	1.0245878	19
8.40	1.59661	3.97504	3.73182E-03	46.20	7.59038	3.92311	8.31	1.0245878	19
8.60	1.58822	4.03197	3.64548E-03	47.17	7.72758	4.06864	8.50	1.0245871	20
8.80	1.58045	4.08832	3.56227E-03	48.11	7.86233	4.21642	8.70	1.0245864	21
9.00	1.57323	4.14412	3.48200E-03	49.01	7.99469	4.36635	8.90	1.0245858	22
9.20	1.56651	4.19943	3.40453E-03	49.89	8.12469	4.51834	9.10	1.0245858	22
9.40	1.56024	4.25430	3.32972E-03	50.75	8.25239	4.67227	9.30	1.0245851	23
9.60	1.55436	4.30877	3.25744E-03	51.57	8.37781	4.82805	9.50	1.0245844	24
9.80	1.54883	4.36288	3.18756E-03	52.37	8.50101	4.98560	9.71	1.0245837	25

S	VELOCITY	WIDTH	DENDIF	THETAO	X COORD	Y COORD	DILUTION	AMDEN	M
10.00	1.54362	4.41666	3.11997E-03	53.15	8.62204	5.14482	9.91	1.0245837	25
10.20	1.53868	4.47016	3.05457E-03	53.89	8.74094	5.30563	10.12	1.0245831	26
10.40	1.53400	4.52340	2.99124E-03	54.62	8.85776	5.46797	10.33	1.0245824	27
10.60	1.52954	4.57642	2.92991E-03	55.32	8.97256	5.63174	10.55	1.0245817	28
10.80	1.52527	4.62923	2.87048E-03	56.00	9.08537	5.79689	10.76	1.0245810	29
11.00	1.52118	4.68187	2.81287E-03	56.66	9.19625	5.96334	10.98	1.0245810	29
11.20	1.51724	4.73436	2.75701E-03	57.30	9.30524	6.13103	11.20	1.0245803	30
11.40	1.51344	4.78672	2.70281E-03	57.91	9.41238	6.29991	11.42	1.0245797	31
11.60	1.50976	4.83896	2.65021E-03	58.51	9.51773	6.46991	11.64	1.0245790	32
11.80	1.50619	4.89111	2.59915E-03	59.09	9.62133	6.64099	11.86	1.0245783	33
12.00	1.50271	4.94318	2.54957E-03	59.65	9.72322	6.81309	12.09	1.0245776	34
12.20	1.49931	4.99519	2.50140E-03	60.20	9.82344	6.98616	12.32	1.0245769	35
12.40	1.49599	5.04714	2.45458E-03	60.72	9.92204	7.16016	12.55	1.0245769	35
12.60	1.49273	5.09906	2.40908E-03	61.24	10.01906	7.33506	12.78	1.0245763	36
12.80	1.48953	5.15095	2.36484E-03	61.73	10.11454	7.51080	13.01	1.0245756	37
13.00	1.48638	5.20282	2.32180E-03	62.21	10.20851	7.68734	13.25	1.0245749	38
13.20	1.48327	5.25467	2.27993E-03	62.68	10.30101	7.86467	13.48	1.0245742	39
13.40	1.48021	5.30653	2.23918E-03	63.14	10.39209	8.04273	13.72	1.0245736	40
13.60	1.47717	5.35839	2.19951E-03	63.58	10.48177	8.22149	13.96	1.0245729	41
13.80	1.47417	5.41026	2.16089E-03	64.01	10.57009	8.40093	14.21	1.0245722	42
14.00	1.47119	5.46214	2.12327E-03	64.42	10.65708	8.58102	14.45	1.0245715	43
14.20	1.46823	5.51405	2.08662E-03	64.83	10.74279	8.76173	14.70	1.0245715	43
14.40	1.46529	5.56598	2.05090E-03	65.22	10.82723	8.94302	14.95	1.0245708	44
14.60	1.46237	5.61794	2.01609E-03	65.60	10.91044	9.12489	15.20	1.0245702	45
14.80	1.45947	5.66994	1.98216E-03	65.98	10.99246	9.30730	15.45	1.0245695	46
15.00	1.45658	5.72197	1.94906E-03	66.34	11.07330	9.49023	15.70	1.0245688	47
15.20	1.45370	5.77404	1.91678E-03	66.69	11.15300	9.67367	15.96	1.0245681	48
15.40	1.45083	5.82616	1.88529E-03	67.03	11.23159	9.85758	16.21	1.0245675	49
15.60	1.44797	5.87832	1.85456E-03	67.37	11.30908	10.04195	16.47	1.0245668	50
15.80	1.44512	5.93052	1.82456E-03	67.69	11.38552	10.22677	16.73	1.0245661	51
16.00	1.44228	5.98278	1.79528E-03	68.01	11.46091	10.41202	17.00	1.0245654	52
16.20	1.43944	6.03508	1.76668E-03	68.32	11.53530	10.59767	17.26	1.0245647	53
16.40	1.43661	6.08744	1.73876E-03	68.62	11.60869	10.78372	17.53	1.0245647	53
16.60	1.43378	6.13984	1.71148E-03	68.92	11.68111	10.97014	17.80	1.0245641	54
16.80	1.43096	6.19231	1.68483E-03	69.20	11.75259	11.15693	18.07	1.0245634	55
17.00	1.42814	6.24482	1.65878E-03	69.48	11.82314	11.34408	18.34	1.0245627	56
17.20	1.42533	6.29739	1.63332E-03	69.76	11.89280	11.53156	18.61	1.0245620	57
17.40	1.42252	6.35002	1.60843E-03	70.02	11.96157	11.71936	18.89	1.0245614	58
17.60	1.41971	6.40270	1.58410E-03	70.28	12.02947	11.90748	19.16	1.0245607	59
17.80	1.41691	6.45543	1.56030E-03	70.54	12.09653	12.09590	19.44	1.0245600	60
18.00	1.41411	6.50823	1.53701E-03	70.78	12.16276	12.28462	19.72	1.0245593	61
18.20	1.41132	6.56108	1.51424E-03	71.03	12.22819	12.47361	20.00	1.0245586	62
18.40	1.40852	6.61399	1.49195E-03	71.26	12.29283	12.66288	20.29	1.0245580	63
18.60	1.40574	6.66695	1.47014E-03	71.49	12.35669	12.85241	20.57	1.0245573	64
18.80	1.40295	6.71997	1.44879E-03	71.72	12.41979	13.04219	20.86	1.0245566	65
19.00	1.40017	6.77305	1.42789E-03	71.94	12.48215	13.23222	21.15	1.0245559	66
19.20	1.39740	6.82618	1.40742E-03	72.16	12.54379	13.42249	21.44	1.0245553	67
19.40	1.39462	6.87937	1.38738E-03	72.37	12.60471	13.61298	21.73	1.0245546	68
19.60	1.39186	6.93262	1.36775E-03	72.58	12.66494	13.80370	22.02	1.0245539	69
19.80	1.38909	6.98592	1.34852E-03	72.78	12.72449	13.99463	22.32	1.0245532	70

S	VELOCITY	WIDTH	DENDIF	THETAD	X COORD	Y COORD	DILUTION	AMDEN	M
20.00	1.38633	7.03928	1.32967E-03	72.98	12.78336	14.18577	22.62	1.0245532	70
20.20	1.38358	7.09269	1.31120E-03	73.17	12.84159	14.37710	22.92	1.0245525	71
20.40	1.38083	7.14616	1.29310E-03	73.36	12.89917	14.56864	23.22	1.0245519	72
20.60	1.37808	7.19969	1.27536E-03	73.55	12.95612	14.76036	23.52	1.0245512	73
20.80	1.37534	7.25327	1.25797E-03	73.73	13.01245	14.95226	23.82	1.0245505	74
21.00	1.37260	7.30691	1.24091E-03	73.91	13.06818	15.14434	24.13	1.0245498	75
21.20	1.36987	7.36060	1.22418E-03	74.08	13.12331	15.33659	24.44	1.0245492	76
21.40	1.36714	7.41434	1.20777E-03	74.26	13.17787	15.52900	24.74	1.0245485	77
21.60	1.36442	7.46814	1.19168E-03	74.42	13.23185	15.72158	25.05	1.0245478	78
21.80	1.36171	7.52199	1.17589E-03	74.59	13.28527	15.91431	25.37	1.0245471	79
22.00	1.35900	7.57590	1.16039E-03	74.75	13.33814	16.10719	25.68	1.0245464	80
22.20	1.35629	7.62986	1.14519E-03	74.91	13.39048	16.30022	26.00	1.0245458	81
22.40	1.35359	7.68387	1.13026E-03	75.07	13.44228	16.49339	26.31	1.0245451	82
22.60	1.35090	7.73793	1.11561E-03	75.22	13.49356	16.68671	26.63	1.0245444	83
22.80	1.34822	7.79205	1.10122E-03	75.37	13.54434	16.88016	26.95	1.0245437	84
23.00	1.34553	7.84621	1.08710E-03	75.51	13.59461	17.07373	27.27	1.0245431	85
23.20	1.34286	7.90043	1.07322E-03	75.66	13.64439	17.26744	27.60	1.0245424	86
23.40	1.34019	7.95470	1.05960E-03	75.80	13.69369	17.46127	27.92	1.0245417	87
23.60	1.33753	8.00902	1.04621E-03	75.94	13.74251	17.65523	28.25	1.0245410	88
23.80	1.33488	8.06339	1.03307E-03	76.08	13.79087	17.84929	28.58	1.0245403	89
24.00	1.33223	8.11782	1.02015E-03	76.21	13.83877	18.04347	28.90	1.0245397	90
24.20	1.32958	8.17229	1.00745E-03	76.34	13.88622	18.23776	29.24	1.0245390	91
24.40	1.32695	8.22681	9.94975E-04	76.47	13.93322	18.43216	29.57	1.0245383	92
24.60	1.32432	8.28138	9.82712E-04	76.60	13.97979	18.62666	29.90	1.0245376	93
24.80	1.32170	8.33600	9.70657E-04	76.72	14.02594	18.82127	30.24	1.0245369	94
25.00	1.31908	8.39067	9.58805E-04	76.85	14.07166	19.01598	30.58	1.0245363	95
25.20	1.31647	8.44538	9.47151E-04	76.97	14.11697	19.21077	30.91	1.0245356	96
25.40	1.31387	8.50015	9.35691E-04	77.08	14.16188	19.40567	31.26	1.0245349	97
25.60	1.31128	8.55496	9.24421E-04	77.20	14.20638	19.60066	31.60	1.0245342	98
25.80	1.30869	8.60982	9.13336E-04	77.32	14.25049	19.79573	31.94	1.0245336	99
26.00	1.30611	8.66473	9.02431E-04	77.43	14.29422	19.99089	32.29	1.0245336	99
26.20	1.30353	8.71969	8.91703E-04	77.54	14.33756	20.18614	32.63	1.0245329	100
26.40	1.30097	8.77470	8.81147E-04	77.65	14.38054	20.38147	32.98	1.0245322	101
26.60	1.29841	8.82975	8.70761E-04	77.75	14.42314	20.57687	33.33	1.0245315	102
26.80	1.29585	8.88486	8.60539E-04	77.86	14.46538	20.77237	33.68	1.0245308	103
27.00	1.29331	8.94000	8.50478E-04	77.96	14.50726	20.96794	34.03	1.0245302	104
27.20	1.29077	8.99520	8.40575E-04	78.07	14.54880	21.16357	34.39	1.0245295	105
27.40	1.28824	9.05044	8.30825E-04	78.17	14.58998	21.35928	34.74	1.0245288	106
27.60	1.28572	9.10573	8.21227E-04	78.26	14.63083	21.55507	35.10	1.0245281	107
27.80	1.28320	9.16107	8.11776E-04	78.36	14.67135	21.75092	35.46	1.0245275	108
28.00	1.28069	9.21645	8.02470E-04	78.46	14.71153	21.94684	35.82	1.0245268	109
28.20	1.27818	9.27188	7.93304E-04	78.55	14.75139	22.14282	36.18	1.0245261	110
28.40	1.27569	9.32736	7.84277E-04	78.64	14.79093	22.33887	36.54	1.0245254	111
28.60	1.27320	9.38288	7.75385E-04	78.74	14.83015	22.53499	36.90	1.0245247	112
28.80	1.27072	9.43845	7.66625E-04	78.82	14.86907	22.73117	37.27	1.0245241	113
29.00	1.26824	9.49406	7.57994E-04	78.91	14.90768	22.92741	37.64	1.0245234	114
29.20	1.26577	9.54972	7.49491E-04	79.00	14.94599	23.12372	38.01	1.0245227	115
29.40	1.26331	9.60543	7.41111E-04	79.09	14.98400	23.32007	38.38	1.0245220	116
29.60	1.26086	9.66118	7.32853E-04	79.17	15.02172	23.51648	38.75	1.0245214	117
29.80	1.25841	9.71698	7.24714E-04	79.25	15.05915	23.71295	39.12	1.0245207	118

S	VELOCITY	WIDTH	DENDIF	THETAD	X COORD	Y COORD	DILUTION	AMDEN	M
30.00	1.25325	9.78339	5.05356E-04	79.31	15.09635	23.90947	39.49	1.0244557	120
30.20	1.24739	9.85278	3.90995E-04	79.36	15.13335	24.10602	39.87	1.0243914	121
30.40	1.24019	9.92780	2.77716E-04	79.40	15.17019	24.30260	40.24	1.0243271	122
30.60	1.23166	10.00857	1.65499E-04	79.43	15.20693	24.49919	40.62	1.0242629	123
30.80	1.22183	10.09526	5.43236E-05	79.44	15.24359	24.69580	41.00	1.0242629	123
31.00	1.21069	10.18810	-5.58331E-05	79.44	15.28024	24.89241	41.37	1.0241986	124
31.20	1.19825	10.28736	-1.64994E-04	79.43	15.31690	25.08902	41.75	1.0241343	125
31.40	1.18449	10.39343	-2.73182E-04	79.40	15.35364	25.28561	42.13	1.0240700	126
31.60	1.16943	10.50671	-3.80422E-04	79.36	15.39050	25.48219	42.50	1.0240057	127
31.80	1.15302	10.62775	-4.86739E-04	79.30	15.42752	25.67874	42.88	1.0239414	128
32.00	1.13525	10.75716	-5.92157E-04	79.23	15.46478	25.87524	43.25	1.0238771	129
32.20	1.11609	10.89570	-6.96704E-04	79.13	15.50232	26.07169	43.62	1.0238129	130
32.40	1.09549	11.04427	-8.00404E-04	79.02	15.54021	26.26807	43.99	1.0237486	131
32.60	1.07340	11.20396	-9.03285E-04	78.89	15.57852	26.46436	44.36	1.0236843	132
32.80	1.04976	11.37609	-1.00537E-03	78.73	15.61734	26.66055	44.73	1.0236200	133
33.00	1.02447	11.56227	-1.10669E-03	78.54	15.65674	26.85663	45.09	1.0235557	134
33.20	0.99746	11.76449	-1.20727E-03	78.32	15.69685	27.05257	45.45	1.0234914	135
33.40	0.96860	11.98522	-1.30714E-03	78.06	15.73777	27.24834	45.81	1.0234271	136
33.60	0.93775	12.22756	-1.40632E-03	77.76	15.77965	27.44391	46.16	1.0233629	137
33.80	0.90472	12.49553	-1.50483E-03	77.40	15.82265	27.63924	46.51	1.0232986	138
34.00	0.86932	12.79435	-1.60270E-03	76.98	15.86698	27.83426	46.85	1.0232343	139
34.20	0.83125	13.13097	-1.69995E-03	76.47	15.91289	28.02892	47.19	1.0231700	140
34.40	0.79017	13.51495	-1.79658E-03	75.85	15.96070	28.22311	47.52	1.0231057	141
34.60	0.74564	13.95981	-1.89260E-03	75.09	16.01083	28.41673	47.84	1.0230414	142
34.80	0.69705	14.48529	-1.98799E-03	74.13	16.06386	28.60959	48.15	1.0229771	143
35.00	0.64359	15.12235	-2.08269E-03	72.88	16.12056	28.80139	48.46	1.0229129	144
35.20	0.58406	15.92184	-2.17657E-03	71.19	16.18207	28.99168	48.75	1.0228486	145
35.40	0.51672	16.97551	-2.26934E-03	68.76	16.25027	29.17967	49.02	1.0227843	146
35.60	0.44052	18.43347	-2.25853E-03	64.99	16.32835	29.36377	49.28	1.0227200	147
35.80	0.35388	20.61572	-2.24777E-03	58.41	16.42206	29.54034	49.52	1.0227200	148
36.00	0.25432	24.36908	-2.23842E-03	43.46	16.54442	29.69788	49.73	1.0227200	149

MAXIMUM HEIGHT OF RISE HAS BEEN REACHED

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May 14, 1973

ADDENDUM

by

Robert C. Y. Koh

for

COMPUTER PROGRAM FOR ROUND BUOYANT JETS INTO STRATIFIED AMBIENT ENVIRONMENTS

by

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Technical Memorandum 69-1 March 1969

The subroutine DEQ was a library subroutine in the IBM 360 system which Caltech had some years ago. The routine as it is used in Ditmars program must be used on the 360 system since it contains Fortran statements not generally used in other systems. DEQ1 and DEQ2 are two entry points into DEQ. The arguments following DEQ1 and DEQ2 are return statement numbers in the main program depending on the outcome of certain computations in DEQ. The subroutine DEQ is now replaced in the Caltech library by a new version MODDEQ.

The routine solves a system of 1st order ordinary differential equations by using the method of Runge-Kutta-Gill for starting and continues by using the Adams-Moulton predictor-corrector formulas. Truncation error can be controlled by specifying its maximum allowed value and the program will change the step size to attempt to stay below the maximum truncation error.

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The calling sequence is

Call DEQ (K, N, T, Y, YDOT, DELT, EBAR)

K	= Control number initially set to 1 by calling program (If K=4, error return due to too large a truncation error.)
N	= number of equations
T	= independent variable
Y	= dependent variable array
YDOT	= derivative array
DELT	= step size (maximum)
EBAR	= maximum allowed truncation error.

A listing of DEQ is attached. By incorporating with Ditmars listing, they constitute the complete program. However, it will not necessarily run on any system due to the branching return options.

DEQ is a routine written by Caltech personnel for the use of Caltech staff and students. Its use by commercial firms for sale is not permitted. Therefore, its use should be limited to the Ditmars program.

There are of course many other differential-equation-solving subroutines in use now to which the Ditmars program could be adopted.

```
      SUBROUTINE DEQ(/K/,N,/I/,Y,YDOT,DELT,EBAR)                        DEQ 0001
      DOUBLE PRECISION TD,TA,TB                                         DEQ 0002
      COMMON/BLOCKA/TA,YDPA(20),YDOTA(20)                               DEQ 0003
      COMMON/BLOCKB/TB,YDPB(20),YDOTB(20)                               DEQ 0004
      DIMENSION YP(20)                                                  DEQ 0005
      DIMENSION YDOT(20)                                                DEQ 0006
      DIMENSION Y(20),          Y DP(20,4),F(20,4)                      DEQ 0007
      DOUBLE PRECISION YDP                                              DEQ 0008
      LOGICAL SIGN                                                      DEQ 0009
      SIGN = .TRUE.                                                     DEQ 0010
      IF(N.GT.20) GO TO 300                                             DEQ 0011
C  INITIALIZE AND SAVE                                                  DEQ 0012
      LSTEP = 0                                                         DEQ 0013
      NU= 1                                                             DEQ 0014
      TIME = T                                                          DEQ 0015
      TD = T                                                            DEQ 0016
      DO 10 I = 1,N                                                     DEQ 0017
   10 YDP(I,1)=Y(I)                                                     DEQ 0018
      IF(DELT.LT.0.) SIGN = .FALSE.                                     DEQ 0019
C FIXED MODE                                                            DEQ 0020
      IF (EBAR.NE.0.) GO TO 20                                          DEQ 0021
      DT = DELT                                                         DEQ 0022
      GO TO 30                                                          DEQ 0023
C VARIABLE MODE                                                         DEQ 0024
   20 CONTINUE                                                          DEQ 0025
      NOSTEP = 4                                                        DEQ 0026
      DT = DELT/FLOAT(NOSTEP)                                           DEQ 0027
      EUP = EBAR                                                        DEQ 0028
      ELO = 0.02*EUP                                                    DEQ 0029
      DTMAX = DELT                                                      DEQ 0030
      DTMIN = DTMAX/FLOAT(2**10)                                        DEQ 0031
      TM = TIME + DTMAX                                                 DEQ 0032
   30 CONTINUE                                                          DEQ 0033
      J = 1                                                             DEQ 0034
      K = 1                                                             DEQ 0035
   35 RETURN                                                            DEQ 0036
C                                                                       DEQ 0037
C                                                                       DEQ 0038
       ENTRY DEQ2(*,*)                                                  DEQ 0039
      IF(LSTEP.NE.0) GO TO 36                                           DEQ 0040
      DO 34 I = 1,N                                                     DEQ 0041
   34 F(I,J) = YDOT(I)                                                  DEQ 0042
      K = 2                                                             DEQ 0043
      RETURN 1                                                          DEQ 0044
   36 CONTINUE                                                          DEQ 0045
      GO TO (105,125,132,187,235,245,255,285),NEXT                      DEQ 0046
C  CONTINUE INTEGRATION                                                 DEQ 0047
      ENTRY DEQ1(*)                                                     DEQ 0048
      IF(EBAR.NE.0.) GO TO 130                                          DEQ 0049
C FIXED MODE                                                            DEQ 0050
C                                                                       DEQ 0051
      IF(J.GT.3) GO TO 120                                              DEQ 0052
      LSTEP = 1                                                         DEQ 0053
      NEXT = 1                                                          DEQ 0054
      K = 1                                                             DEQ 0055
  105  IF (LSTEP .EQ. 5) GO TO 110                                      DEQ 0056
      CALL GILL(N,LSTEP,DT,T,Y,YDOT,YDP,TD)                             DEQ 0057
      RETURN 1                                                          DEQ 0058
  110 J = J + 1                                                         DEQ 0059
      DO 115 I = 1,N                                                    DEQ 0060
```

```
  115 F(I,J) = YDOT(I)                                                  DEQ 0061
      RETURN 2                                                          DEQ 0062
C CALL ADAM                                                             DEQ 0063
  120 LSTEP = 5                                                         DEQ 0064
      NEXT = 2                                                          DEQ 0065
  125 IF(LSTEP .EQ. 7) GO TO 128                                        DEQ 0066
      CALL ADAM(N,LSTEP,DT,T,Y,YDOT,YDP,YP,F,TD)                        DEQ 0067
      RETURN 1                                                          DEQ 0068
  128 DO 129 M = 1,3                                                    DEQ 0069
      DO 129 I = 1,N                                                    DEQ 0070
  129 F(I,M) = F(I,M+1)                                                 DEQ 0071
      DO 1280 I = 1,N                                                   DEQ 0072
 1280 F(I,4) = YDOT(I)                                                  DEQ 0073
      RETURN 2                                                          DEQ 0074
C VARIABLE MODE                                                         DEQ 0075
C                                                                       DEQ 0076
  130 CONTINUE                                                          DEQ 0077
          TIME = T                                                      DEQ 0078
      IF(J.GT.3) GO TO 170                                              DEQ 0079
      LSTEP = 1                                                         DEQ 0080
      NEXT = 3                                                          DEQ 0081
      K = 1                                                             DEQ 0082
  132  IF (LSTEP .EQ. 5) GO TO 135                                      DEQ 0083
      CALL GILL(N,LSTEP,DT,T,Y,YDOT,YDP,TD)                             DEQ 0084
      RETURN 1                                                          DEQ 0085
  135 J = J + 1                                                         DEQ 0086
      DO 137 I = 1,N                                                    DEQ 0087
  137 F(I,J) = YDOT(I)                                                  DEQ 0088
      TIME = T                                                          DEQ 0089
      GO TO (140,150,160,170),J                                         DEQ 0090
C 2A  J = 1                                                             DEQ 0091
C                                                                       DEQ 0092
  140 CONTINUE                                                          DEQ 0093
      IF(SIGN ) GO TO 142                                               DEQ 0094
      IF(TIME+.01*DT .LE. TM) GO TO 145                                 DEQ 0095
      RETURN 2                                                          DEQ 0096
  142 CONTINUE                                                          DEQ 0097
      IF(TIME +.01*DT .GE. TM) GO TO 145                                DEQ 0098
      RETURN 2                                                          DEQ 0099
  145 TM = TIME + DTMAX                                                 DEQ 0100
      K = 3                                                             DEQ 0101
      RETURN 1                                                          DEQ 0102
C  J=2                                                                  DEQ 0103
C                                                                       DEQ 0104
  150 CALL SAVE(MU,N,TIME,Y,YDOT,TD)                                    DEQ 0105
      GO TO 140                                                         DEQ 0106
C  J = 3                                                                DEQ 0107
C                                                                       DEQ 0108
  160 L = 1                                                             DEQ 0109
      RETURN 2                                                          DEQ 0110
C  J.GE. 3                                                              DEQ 0111
C                                                                       DEQ 0112
  170 CONTINUE                                                          DEQ 0113
      GO TO (180,250,280),L                                             DEQ 0114
C 3A                                                                    DEQ 0115
C                                                                       DEQ 0116
  180 MU = -MU                                                          DEQ 0117
C 3C                                                                    DEQ 0118
C                                                                       DEQ 0119
  185 CONTINUE                                                          DEQ 0120
```

```
      CALL SAVE(MU,N,TIME,Y,YDOT,TD)                                    DEQ 0121
      K = 1                                                             DEQ 0122
      LSTEP = 5                                                         DEQ 0123
      NEXT = 4                                                          DEQ 0124
  187  IF (LSTEP .EQ. 7) GO TO 190                                      DEQ 0125
      CALL ADAM(N,LSTEP,DT,T,Y,YDOT,YDP,YP,F,TD)                        DEQ 0126
      RETURN 1                                                          DEQ 0127
C                                                                       DEQ 0128
C COMPUTE E(N+1)                                                        DEQ 0129
  190 EN1 = 0.0                                                         DEQ 0130
      TIME = T                                                          DEQ 0131
      DO 195 M = 1,5                                                    DEQ 0132
      DO 195 I = 1,N                                                    DEQ 0133
  195 F(I,M) = F(I,M+1)                                                 DEQ 0134
      DO 196 I = 1,N                                                    DEQ 0135
  196 F(I,4) = YDOT(I)                                                  DEQ 0136
      DO 200 I = 1,N                                                    DEQ 0137
      DI = AMAX1(ABS(Y (I)),.001)                                       DEQ 0138
      EN1 = AMAX1 (ABS(YP(I)-Y (I))/(14.0*DI),EN1)                      DEQ 0139
  200 CONTINUE                                                          DEQ 0140
C TEST E(N+1) .GE.EPSIL                                                 DEQ 0141
C                                                                       DEQ 0142
      IF (EN1.GE.EUP) GO TO 220                                         DEQ 0143
      IODD = ABS((TIME-TH)/DT) + .0005                                  DEQ 0144
      IF(MOD(IODD,2) .NE. 0) GO TO 210                                  DEQ 0145
C EVEN                                                                  DEQ 0146
      IF (EN1.GE.FL0) GO TO 210                                         DEQ 0147
C     IF (2.0*DT.GT.DTMAX) GO TO 210                                    DEQ 0148
       IF (ABS(2.0*DT) .GT. ABS(DTMAX)) GO TO 210                       DEQ 0149
      NU = 0                                                            DEQ 0150
      L = 2                                                             DEQ 0151
      GO TO 140                                                         DEQ 0152
C SH                                                                    DEQ 0153
  210 L = 1                                                             DEQ 0154
      GO TO 140                                                         DEQ 0155
C EN1 .GE. EPSIL                                                        DEQ 0156
C                                                                       DEQ 0157
C 220 IF (.25*DT.GE.DTMIN) GO TO 230                                    DEQ 0158
  220   IF (ABS(.25*DT) .GE. ABS(DTMIN)) GO TO 230                      DEQ 0159
C GO TO ERROR                                                           DEQ 0160
      K = 4                                                             DEQ 0161
      RETURN 1                                                          DEQ 0162
C REDUCE STEP                                                           DEQ 0163
  230 CONTINUE                                                          DEQ 0164
      NOSTEP = NOSTEP*4                                                 DEQ 0165
      DT = DELT/FLOAT(NOSTEP)                                           DEQ 0166
      CALL RESTOR(-ML,N,TIME,Y,YDOT,TD)                                 DEQ 0167
      T = TIME                                                          DEQ 0168
  232 NEXT = 5                                                          DEQ 0169
      LSTEP = 1                                                         DEQ 0170
      K = 1                                                             DEQ 0171
      IJ = 1                                                            DEQ 0172
      DO 233 I = 1,N                                                    DEQ 0173
      F(I,1) = YDOT(I)                                                  DEQ 0174
  233 CONTINUE                                                          DEQ 0175
  235   IF (LSTEP .EQ. 5) GO TO 240                                     DEQ 0176
      CALL GILL(N,LSTEP,DT,T,Y,YDOT,YDP,TD)                             DEQ 0177
      RETURN 1                                                          DEQ 0178
  240 CONTINUE                                                          DEQ 0179
      IJ = IJ + 1                                                       DEQ 0180
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      DO 242 I = 1,N                                                    DEQ 0181
  242 F(I,IJ) = YDOT(I)                                                 DEQ 0182
      LSTEP = 1                                                         DEQ 0183
      IF(IJ.LE. 3) GO TO 235                                            DEQ 0184
      TIME = T                                                          DEQ 0185
      CALL SAVE(-MU,N,TIME,Y,YDOT,TD)                                   DEQ 0186
      NEXT = 6                                                          DEQ 0187
      LSTEP = 5                                                         DEQ 0188
      K = 1                                                             DEQ 0189
  245  IF (LSTEP .EQ. 7) GO TO 248                                      DEQ 0190
      CALL ADAM(N,LSTEP,DT,T,Y,YDOT,YDP,YP,F,TD)                        DEQ 0191
      RETURN 1                                                          DEQ 0192
  248 CONTINUE                                                          DEQ 0193
      DO 243 M = 1,3                                                    DEQ 0194
      DO 243 I = 1,N                                                    DEQ 0195
  243 F(I,M) = F(I,M+1)                                                 DEQ 0196
      DO 244 I = 1,N                                                    DEQ 0197
  244 F(I,4) = YDOT(I)                                                  DEQ 0198
      GO TO 185                                                         DEQ 0199
C 4A                                                                    DEQ 0200
  250 MU = -MU                                                          DEQ 0201
      TIME = T                                                          DEQ 0202
      CALL SAVE(MU,N,TIME,Y,YDOT,TD)                                    DEQ 0203
      LSTEP = 5                                                         DEQ 0204
      NEXT = 7                                                          DEQ 0SOV
      K = 1                                                             DEQ 0206
  255 IF(LSTEP .EQ. 7) GO TO 260                                        DEQ 0207
      CALL ADAM(N,LSTEP,DT,T,Y,YDOT,YDP,YP,F,TD)                        DEQ 0208
      RETURN 1                                                          DEQ 0209
C COMPUTE E(N+1)                                                        DEQ 0210
C                                                                       DEQ 0211
  260 EN1 = 0.                                                          DEQ 0212
      TIME = T                                                          DEQ 0213
      DO 265 M = 1,3                                                    DEQ 0214
      DO 265 I = 1,N                                                    DEQ 0215
  265 F(I,M) = F(I,M+1)                                                 DEQ 0216
      DO 266 I = 1,N                                                    DEQ 0217
  266 F(I,4) = YDOT(I)                                                  DEQ 0218
      DO 270 I = 1,N                                                    DEQ 0219
      DI = AMAX1(ABS(Y (I)),.001)                                       DEQ 0220
C     EN1 = AMAX1(ABS(YP(I)-Y (I))/(14.0*DT),EN1)                       DEQ 0221
      EN1 = AMAX1(ABS(YP(I)-Y (I))/(14.0*DI),EN1)                       DEQ 0222
  270 CONTINUE                                                          DEQ 0223
      IF (EN1.GE.ELO) GO TO 210                                         DEQ 0224
      NU = NU+1                                                         DEQ 0225
      IF (NU.LT.2) RETURN  2                                            DEQ 0226
      NOSTEP = NOSTEP/2                                                 DEQ 0227
      DT = DELT/FLOAT(NOSTEP)                                           DEQ 0228
      TIME = T                                                          DEQ 0229
      CALL SAVE(MU,N,TIME,Y,YDOT,TD)                                    DEQ 0230
      II= 0                                                             DEQ 0231
      L = 3                                                             DEQ 0232
      GO TO 140                                                         DEQ 0233
C                                                                       DEQ 0234
C 4B                                                                    DEQ 0235
  280 LSTEP = 1                                                         DEQ 0236
      NEXT = 8                                                          DEQ 0237
      K = 1                                                             DEQ 0238
  285  IF (LSTEP .EQ. 5) GO TO 290                                      DEQ 0239
      CALL GILL(N,LSTEP,DT,T,Y,YDOT,YDP,TD)                             DEQ 0240
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      RETURN 1                                                          DEQ 0241
  290 II=II+1                                                           DEQ 0242
      DO 295 I = 1,N                                                    DEQ 0243
  295 F(I,II) = YDOT(I)                                                 DEQ 0244
      TIME = T                                                          DEQ 0245
      IF(II.GT.5) GO TO 210                                             DEQ 0246
      GO TO 140                                                         DEQ 0247
  300 CONTINUE                                                          DEQ 0248
      WRITE(6,510) N                                                    DEQ 0249
  510 FORMAT(//1X28HORDER OF EQUATIONS EXCEED 20   I5//)                DEQ 0250
      STOP                                                              DEQ 0251
      END                                                               DEQ 0252
      SUBROUTINE RESTOR(MU,N,TIME,YDP,YDOT,TD)                          REST0001
      DOUBLE PRECISION TD,TA,TB                                         REST0002
      COMMON/BLOCKA/TA,YDPA(20),YDOTA(20)                               REST0003
      COMMON/BLOCKB/TB,YDPB(20),YDOTB(20)                               REST0004
      DIMENSION YDP(20),YDOT(20)                                        REST0005
      IF (MU.GT.0) GO TO 30                                             REST0006
      DO 20 I = 1,N                                                     REST0007
      YDP(I) = YDPB(I)                                                  REST0008
   20 YDOT(I) = YDOTB(I)                                                REST0009
      TD = TB                                                           REST0010
      TIME = TB                                                         REST0011
      RETURN                                                            REST0012
C MU.GT.0                                                               REST0013
   30 CONTINUE                                                          REST0014
      DO 40 I = 1,N                                                     REST0015
      YDP(I) = YDPA(I)                                                  REST0016
   40 YDOT(I) = YDOTA(I)                                                REST0017
      TD = TA                                                           REST0018
      TIME = TA                                                         REST0019
      RETURN                                                            REST0020
      ENTRY SAVE(MU,N,TIME,YDP,YDOT,TD)                                 REST0021
      IF (MU.GT.0) GO TO 70                                             REST0022
      TB = TD                                                           REST0023
      DO 60 I = 1,N                                                     REST0024
      YDPB(I) = YDP(I)                                                  REST0025
   60 YDOTB(I) = YDOT(I)                                                REST0026
      RETURN                                                            REST0027
   70 CONTINUE                                                          REST0028
      TA = TD                                                           REST0029
      DO 80 I = 1,N                                                     REST0030
      YDPA(I) = YDP(I)                                                  REST0031
   80 YDOTA(I) = YDOT(I)                                                REST0032
      RETURN                                                            REST0033
      END                                                               REST0034
      SUBROUTINE ADAM (N,LSTEP,DT,T,Y,YDOT,YDP,YP,F,TD)                 ADAM0001
      DOUBLE PRECISION DP                                               ADAM0002
      DIMENSION F(20,4),YP(20),Y(20)                                    ADAM0003
      DIMENSION YDOT(20)                                                ADAM0004
      DOUBLE PRECISION TD                                               ADAM0005
      DOUBLE PRECISION YDP(20,2),YDC(20)                                ADAM0006
      IF (LSTEP.EQ.4 ) GO TO 50                                         ADAM0007
      DO 20 I = 1,N                                                     ADAM0008
       DP       = Y(I)+(DT/24.00)*(55.0D0*F(I,4)-59.0D0*F(I,3)+37.0D0*F( ADAM0009
     1 I,2)-9.0D0*F(I,1))                                               ADAM0010
      YDP(I,1) = DP                                                     ADAM0011
      YP(I) = Y(I)                                                      ADAM0012
      SP = DP                                                           ADAM0013
      D1 = DP-SP                                                        ADAM0014
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      Y(I) = DP+D1                                                          ADAM0015
   20 CONTINUE                                                              ADAM0016
      LSTEP = LSTEP+1                                                       ADAM0017
      TD = TD + DT                                                          ADAM0018
      T = TD                                                                ADAM0019
      RETURN                                                                ADAM0020
   30 CONTINUE                                                              ADAM0021
      DO 40 I = 1,N                                                         ADAM0022
       DP    =YP(I)+(DT/24.000)*(9.000*YDOT(I)+19.000*F(I,4)-5.000*F(I,3   ADAM0023
     1) + F(I,2))                                                           ADAM0024
      YP(I) = Y(I)                                                          ADAM0025
      YDC(I) = DP                                                           ADAM0026
      SP = DP                                                               ADAM0027
      D1 = DP-SP                                                            ADAM0028
      Y(I) = DP+D1                                                          ADAM0029
   40 CONTINUE                                                              ADAM0030
      LSTEP = LSTEP+1                                                       ADAM0031
      RETURN                                                                ADAM0032
      END                                                                   ADAM0033
      SUBROUTINE GILL(N,LSTEP,DT,T,Y,YDOT,YDP,TD)                           GILL0001
      DOUBLE PRECISION DP                                                   GILL0002
      DOUBLE PRECISION YDP                                                  GILL0003
      DOUBLE PRECISION TD,T1                                                GILL0004
      DOUBLE PRECISION K                                                    GILL0006
      DIMENSION C(3),B(3),Q(20),YDP(20,4),Y(20),YDOT(20  )                  GILL0005
      DOUBLE PRECISION B,C                                                  GILL0007
      DATA B,C/0.00, .5857864380D0, 3.41421356200, 0.,                      GILL0008
     1  .1213203430, -4.121320343 /                                         GILL0009
      IF (LSTEP.GT.1) GO TO 25                                              GILL0010
      T1 = TD                                                               GILL0011
      DO 20 I = 1,N                                                         GILL0012
      K = DT*YDOT(I)                                                        GILL0013
      YDP(I,LSTEP) = Y(I) + .5D0*K                                          GILL0014
      DP = YDP(I,LSTEP)                                                     GILL0015
      SP = DP                                                               GILL0016
      D1 = DP-SP                                                            GILL0017
      Y(I) = DP+D1                                                          GILL0018
      Q(I) = K                                                              GILL0019
   20 CONTINUE                                                              GILL0020
      TD = TD + .5*DT                                                       GILL0021
      T = TD                                                                GILL0022
      LSTEP = LSTEP+1                                                       GILL0023
      RETURN                                                                GILL0024
   25 IF(LSTEP.GE.4) GO TO 50                                               GILL0025
      DO 30 I = 1,N                                                         GILL0026
      K = DT*YDOT(I)                                                        GILL0027
      YDP(I,LSTEP) = YDP(I,LSTEP-1)+.5D0*B(LSTEP)*(K-Q(I))                  GILL0028
      DP = YDP(I,LSTEP)                                                     GILL0029
      SP = DP                                                               GILL0030
      D1 = DP-SP                                                            GILL0031
      Y(I) = DP+D1                                                          GILL0032
      Q(I) = B(LSTEP)*K+C(LSTEP)*Q(I)                                       GILL0033
   30 CONTINUE                                                              GILL0034
      LSTEP = LSTEP+1                                                       GILL0035
      IF (LSTEP.EQ.4)TD= T1+DT                                              GILL0036
      T = TD                                                                GILL0037
      RETURN                                                                GILL0038
   50 DO 60 I = 1,N                                                         GILL0039
      K = DT*YDOT(I)                                                        GILL0040
      YDP(I,LSTEP) = YDP(I,LSTEP-1)+(K*.5D0-Q(I))/3.000                     GILL0041
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      DP = YDP(I,LSTEP)                                          GILL0042
      SP = DP                                                    GILL0043
      D1 = DP-SP                                                 GILL0044
      Y(I) = DP+D1                                               GILL0045
   60 CONTINUE                                                   GILL0046
      LSTEP = LSTEP+1                                            GILL0047
      RETURN                                                     GILL0048
      END                                                        GILL0049
```